Chapter 1

NUCLEAR REACTIONS IN STARS: THEORETICAL AND EXPERIMENTAL ASPECTS

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1. Introduction

PARLER DES TYPES DE REACTION: TRANSFER, CAP. Nuclear reactions are known to play a fundamental role in the evolution of stars [**?**, **?**]. They are responsible for the nucleosynthesis, i.e. the formation of the elements in the Universe. Big-Bang nucleosynthesis essentially provides protons ($\sim 75\%$) and α particles ($\sim 25\%$). Small amounts of heavier elements are also produced, up to ^{12}C [**?**]. The primordial nucleosynthesis is followed by the formation of early starts where elements up to Fe are produced. Heavier elements are then synthesized by various processes, such as neutron capture, and explosive events in supernovae [**?**, **?**]. Astrophysical scenarios and stellar models are discussed in many books (see for example Refs. [**?**, **?**, **?**, **?**]) and review articles (see for example Refs. [**?**, **?**, **?**]).

The role of nuclear physics in astrophysics is fundamental, and this discipline is referred to as nuclear astrophysics. Many observational properties find their origin in nuclear physics. (*i*) A famous example is the 0_2^+, known as the Hoyle state [**?**], which was predicted from the observed ^{12}C abundance in the Universe, and then found experimentally. The formation of ^{12}C is currently well understood from the triple α process. (*ii*) From the observed abundances of the elements, a gap between masses 5 and 8 is explained by the particle instability of ^{5}He and ^{5}Li. (*iii*) In the high-mass region of the abundance distribution, peaks are clearly observed, and are explained by the existence of magic numbers in nuclear physics. Magic nuclei are known to be strongly bound, and therefore difficult to destroy by photodissociation. (*iv*) The abundance distribution also presents an "even-odd" effect, even nuclei being more abundant; again the origin of this effect stems from nuclear binding energies since odd-mass nuclei are less bound and therefore more fragile than even-mass nuclei.

Stellar models require many nuclear inputs. A huge number (up to several thousands) of reaction rates, involving charged particles (protons and alphas) and neutrons, are needed in nucleosynthesis models. A challenge for nuclear physicists is to determine the cross sections at stellar energies, which are in general much lower than the Coulomb barrier. Except in a few cases, direct measurements at these low energies (referred to as the "Gamow peak") are not possible, since the cross sections are too low to be measured in the laboratory. A theoretical support is then necessary to complement the data, and in particular to extrapolate them down to stellar energies. Nuclear astrophysics is a field where the complementarity between experiment and theory is crucial.

Experimental techniques have been strongly developed in the last decades, with two main objectives: going to energies as low as possible, and investigating reactions involving radioactive nuclei. Many important reactions involve short-lived nuclei (such as ^{7}Be, ^{8}Li, ^{13}N, ^{18}F, etc.) and can be studied with radioactive beams only. Direct measurements also take benefit from underground facilities, such as LUNA [], where the background is strongly reduced and allows precise measurements, even at very low energies where the cross sections can be as low as $10^{??}$ barn.

Owing to the experimental difficulties associated with direct measurements, several indirect techniques have been developed, such as Coulomb breakup [], the Asymptotic Normalization Constant (ANC, see Ref. []) method, or the Trojan Horse [] method. The advantages of these indirect approaches is to circumvent the smallness of the cross sections. However they require a precise theoretical modeling for the determination of the relevant

cross sections from the data.

The determination of reaction rates is of course based on the scattering theory. Various models are being used in the literature. Owing to the low energies relevant in nuclear astrophysics, and to the low level densities, the optical model (also referred to as the "potential model") can be used for capture reactions [**?**]. Heavy-ion fusion reactions are also described by this model, even if different variants exit [**?**]. An extension of the standard optical model is the Distorted Wave Born Approximation (DWBA, see Ref. [**?**]), used to describe transfer processes such as (α,n) or (α,p). Important developments have been performed in the framework of microscopic models, which present an important predictive power since they only rely on a nucleon-nucleon interaction. Solving a many-body Schrödinger equation for scattering states is however a difficult task, and the cluster approximation [**?**] is used in most calculations. However recent works succeeded to address, in a microscopic theory, the ^{2}H(^{2}H,γ)^{4}He [**?**], ^{3}He(d,p)^{4}He [**?**] and ^{3}He(α,γ)^{7}Be [**?**] reactions without the cluster approximation. These calculations are highly computer demanding, and are currently limited to low-mass systems.

The models discussed above are, in principle, independent of experimental data. In practice, available data are used to test the model, and/or to tune some important parameter. In contrast, the *R*-matrix theory [] relies on the existence of data, but is an efficient tool to analyze reactions of astrophysical interest. The cross sections can be parametrized by a small number of real, energy-independent, parameters. This fitting procedure requires the availability of experimental data, but in general allows a reliable extrapolation down to stellar energies. The *R*-matrix theory deals with capture, transfer and elastic scattering on an equal footing. As a general statement, the *R*-matrix is efficient to describe any process involving continuum states (an recent example is the ^{12}N and ^{12}B beta decays to three-α states [**?**]). This property is often used, for example, by measuring elastic cross sections to constrain the parameters, and to improve the accuracy of the low-energy extrapolations. In practice, the *R*-matrix theory is limited to the low-energy region, where the level density is limited (typically up to a few levels per MeV).

When the mass increases the level density is in general too large for cluster models or for an *R*-matrix approach. In that situation, the cross section essentially depends on properties of the compound nucleus. Here the shell-model approach [**?**] provides information on resonance properties such as energies, spins, widths, etc. An extension to the continuum has been developed [**?**]. The Hauser-Feshbach formalism [**?**] is also widely used in high-mass systems, with a large level density.

A specificity of nuclear astrophysics is to require a huge number of reaction rates. These reaction rates are obtained from various sources, experimental as well as theoretical. The link between nuclear physics and astrophysics is then performed by compilations, where the authors provide an evaluation of the available data, and recommended reaction rates. The first compilations were performed by the Caltech group [**?**,**?**], and then improved in various ways: evaluation of uncertainties, improved numerical treatment, update of experimental data, etc. Some compilations address specific reactions, such as Bib-Bang nucleosynthesis [**?**, **?**, **?**] or solar fusion reactions [**?**], but other works cover a wider range [**?**, **?**]. Most reactions play a minor role in stellar models. However a few reactions, such a ^{12}C(α,γ)^{16}O or ^{18}F(p,α)^{15}O are crucial, and need a special attention.

Nuclear astrophysics is not limited to reactions: several additional nuclear inputs are

necessary in stellar models. Nuclear masses define the nucleosynthesis in the *r* and *s* processes. Beta-decay rates are important, as they may compete with capture reactions. Electron-capture [] and neutrino-induced reactions [] also play a role in specific stellar environments.

In this review, we discuss the current status of theoretical descriptions, as well as of experimental developments, in nuclear astrophysics. We essentially focus on charged-particle induced reactions, which represent one of the main inputs in stellar evolution. In section 2., we present an overview of the different processes in the nucleosynthesis. Section 3. addresses specific nuclear inputs; in particular we discuss the calculation of the reaction rates. In section 4. we briefly describe some theoretical models used for nuclear reactions. Section 5. is devoted to experiment; we discuss direct as well as indirect techniques, and recent developments of radioactive beams. In section 6., we address a few reactions, which are particularly important. The conclusion and outlook are presented in Section 7.

2. Stellar nucleosynthesis

2.1. Principles

2.2. H burning

2.3. He burning

2.4. s and r processes

2.5. explosive burning

2.6. fusion reactions

3. Overview of nuclear inputs in astrophysics

3.1. Reaction rates

3.2. masses

3.3. beta decay

3.4. neutrino induced reactions

3.5. fission barriers

4. Theoretical models for reactions

4.1. Scattering wave functions

In this section, we present an outline of the reaction theory needed for nuclear astrophysics. In particular we are dealing with low energies, around and below the Coulomb barrier. Our goal is to modelize different processes, such as transfer or capture reactions, by various theories. They will be described in this section. We start from a general formalism of the

reaction theory, and then apply it to different models. This outline is of course very brief, and we refer to textbooks [**?**, **?**, **?**, **?**, **?**] for a detailed presentation.

The main goal is to solve the Schrödinger equation

$$H\Psi_{0k} = E\Psi_{0k} \tag{4.1}$$

for positive energies E. The scattering wave function Ψ_{0k} is characterized by the entrance channel 0 (set of quantum numbers of the colliding nuclei). The description of Ψ_{0k} depends on the model, and on the assumptions concerning the Hamiltonian. In the optical model [**?**], the internal structure of the colliding nuclei is neglected, and the wave function simply depends on the relative coordinate. In microscopic cluster models [**?**, **?**, **?**] and their ab-intro extensions [**?**], the wave function depends on all nucleon coordinates, and is totally antisymmetrized to account for the Pauli principle.

If the short-range properties of the scattering wave functions strongly depend on the adopted model, the analytical expression at large nucleus-nucleus distances is common to all approaches. Let us consider the Hamiltonian of each nucleus H_i $(i = 1,2)$. The associated Schrödinger equation reads

$$H_i|\phi_i^\alpha\rangle = E_i^\alpha|\phi_i^\alpha\rangle, \tag{4.2}$$

where index α represents all quantum numbers of the individual nuclei (spin and projection, parity, isospin, etc.). Energies E_i^α can be negative or positive, and $|\phi_i^\alpha\rangle$ are the individual wave functions normalized as

$$\langle\phi_i^\alpha|\phi_{i'}^\alpha\rangle = \delta_{ii'}. \tag{4.3}$$

When the nucleus-nucleus relative coordinate r is large, the asymptotic behaviour of the scattering wave function (4.1) is given by

$$\Psi_{0k} \xrightarrow[r\to\infty]{} \sum_\alpha \left(e^{ik_0\cdot r}\delta_{\alpha 0} + f_{\alpha 0}(\theta)\frac{e^{ik_\alpha r}}{r}\right)\phi_1^\alpha\phi_2^\alpha, \tag{4.4}$$

where k_α is the wave number in channel α, and $f_{\alpha 0}(\theta)$ is the scattering amplitude depending on the scattering angle θ. In (4.4) we have neglected the Coulomb interaction for the sake of simplicity. Including this interaction introduces distortion effects but does not change the physical interpretation (see detail in Ref. [**?**]). Notice that the asymptotic expression (4.4) can be defined with different normalization factors. This does not affect the scattering amplitudes, but needs to be specified when using scattering wave functions in matrix elements.

Let us first expand to total wave function Ψ_{0k} over the different channels α as

$$\Psi_{0k} = \sum_\alpha \Psi_{0k}^\alpha. \tag{4.5}$$

Each component can be written as

$$\Psi_{0k}^\alpha = \sum_\ell \Psi_{0k,\ell}^\alpha Y_\ell^0(\Omega_k), \tag{4.6}$$

where the wave vector k is assumed to be in the z direction. Since the total angular momentum JM and parity π are good quantum numbers, (4.6) is expanded as

$$\Psi_{0k,\ell}^\alpha = \sum_{JI}\langle I_1^\alpha I_2^\alpha \nu_1^\alpha \nu_2^\alpha|I\nu\rangle\langle\ell I 0\nu|JM\rangle\Psi_{\alpha\ell I}^{JM}. \tag{4.7}$$

This partial-wave expansion is useful at low energies, where a small number of J values contribute to the wave function, and hence to the cross section. In (4.7), I is the channel spin, and stems from the coupling of I_1^α and I_2^α. For the sake of clarity, the entrance channel index 0 and the wave number k are implied in $\Psi^{JM}_{\alpha\ell I}$. Here also, expansion (4.7) does not depend on the model. At large distances a partial wave tends to

$$\Psi_c^{JM} \xrightarrow[r\to\infty]{} A_c \frac{1}{r}\left(I_\ell(k_0 r) - U_{0c}^{J\pi} O_\ell(k_c r)\right)\varphi_c^{JM}, \tag{4.8}$$

where $I_\ell(x)$ and $O_\ell(x)$ are the ingoing and outgoing Coulomb functions [**?**], A_c a normalization factor and φ_c^{JM} is the channel wave function

$$\varphi_c^{JM} = \left[\left[\phi_1^\alpha \otimes \phi_2^\alpha\right]^I \otimes Y_\ell(\Omega_r)\right]^{JM}. \tag{4.9}$$

Index c stands for $c = (\alpha\ell I)$. In (4.8), $U^{J\pi}$ is the collision matrix (or scattering matrix), sometimes referred to as $S^{J\pi}$ in the literature), which provides elastic, inelastic or transfer cross sections. The calculation of the collision matrix depends on the model. The next subsections are devoted to some models commonly used in nuclear astrophysics.

4.2. Models

4.2.1. The optical model

The main idea of the optical model is to simplify the Schödinger equation as much as possible, and in particular to simulate excited channels by an appropriate choice of the potential. In this way, the nucleus-nucleus interaction is reduced to a radial potential, depending on the relative coordinate only. Formally the Feshbach theory [**?**] shows that excited configurations can be taken into account by an energy-dependent, complex and non-local potential. The imaginary part simulates the absorption to inelastic channels, which are not explicitly included in the calculation. As the number of open channels increases with energy, the amplitude of the absorption term follows this property. At very low energies, typical of astrophysical interest, the elastic channel is often the only open channel, and the nucleus-nucleus potential can be approximated by a real potential.

In practice the original Feshbach theory is difficult to apply. It is however the starting point for various approximations. In the double folding model [**?**, **?**], the Hamiltonian of the system reads

$$H = H_1 + H_2 + T_R + \sum_{i=1}^{A_1}\sum_{j=1}^{A_2} v_{NN}(r_i - r_j), \tag{4.10}$$

where H_i are the internal hamiltonian, T_R is the relative kinetic energy, and $v_{NN}(r)$ an effective nucleon-nucleon potential. Typical examples are the M3Y [**?**] and São Paulo potentials (SPP, see Refs. [**?**, **?**]). The M3Y potential is fitted on the T matrix obtained with the Reid potential. The SPP interaction is simpler and has been fitted on several systems; the Pauli principle is approximated by a non-local term. Various improvements have been proposed to include a density dependence in the NN interaction [**?**] (see a review in Ref. [**?**]).

If we neglect the antisymmetrization between the colliding nuclei, the wave function of the system is expanded as

$$\Psi^{JM\pi} = \sum_c g_c^{J\pi}(r)\varphi_c^{JM\pi}, \tag{4.11}$$

where excited channels can be, in principle, taken into account. The relative functions $g_c^{J\pi}(r)$ provide the scattering matrix. Inserting expression (4.11) in the Schrödinger equation provides a coupled-channel system

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}g_c^{J\pi}(r) + \sum_{c'} V_{cc'}^{J\pi}(r)g_{c'}^{J\pi}(r) = (E - E_1^c - E_2^c)g_c^{J\pi}(r), \tag{4.12}$$

where the coupling potentials are obtained from

$$V_{\alpha\alpha'}(r) = \langle \phi_1^\alpha \phi_2^\alpha | \sum_{i=1}^{A_1}\sum_{j=1}^{A_2} v_{NN}(r - \xi_i + \xi_j) | \phi_1^{\alpha'} \phi_2^{\alpha'} \rangle, \tag{4.13}$$

and additional algebraic coefficients [**?**]. Potential (4.13) is written in a more elegant form

$$V_{\alpha\alpha'}(r) = \iint dr_1 dr_2 v_{NN}(r - r_1 + r_2)\rho_1^{\alpha\alpha'}(r_1)\rho_2^{\alpha\alpha'}(r_2), \tag{4.14}$$

where $\rho_k^{\alpha\alpha'}$ are the nuclear densities

$$\rho_k^{\alpha\alpha'}(r) = \frac{1}{A_k}\langle \phi_k^\alpha | \sum_{i=1}^{A_k} \delta(r - \xi_i) | \phi_k^{\alpha'} \rangle. \tag{4.15}$$

The densities are usually taken from experiment or from theoretical calculations. Equation (4.14) is known as the double-folding potential [**?**]. The calculation is usually performed by using Fourier transforms. In most calculations, a single channel is included. As the M3Y and SP interactions are real, a phenomenological imaginary term is introduced as

$$W(r) = N_I V(r). \tag{4.16}$$

The M3Y and SP potentials, combined with experimental densities, are equivalent to the Buck potential [**?**] for $\alpha - \alpha$ scattering [**?**, **?**].

In nuclear astrophysics, the nucleus-nucleus potential is usually real. It can be obtained, either from the folding method [**?**] or by assuming a specific shape, such as a Woods-Saxon or a Gaussian potential [**?**]. The parameters are fitted on important properties of the system, such as binding energies, resonance energies, or low-energy phase shifts. The technique is clearly limited to systems where experimental data are available. Also, the high level densities of heavy nuclei are not well adapted.

When the potential is determined, the main issue is to solve the coupled-channel system (4.12). Essentially two approaches are used in the literature: discretization methods, such as the Numerov algorithm [**?**, **?**] or the *R*-matrix method [**?**].

4.2.2. Microscopic cluster theories

Microscopic models are based on fundamental principles of quantum mechanics, such as the treatment of all nucleons, with exact antisymmetrization of the wave functions. The Hamiltonian of a A-nucleon system is

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j=1}^{A} V_{ij}, \tag{4.17}$$

where T_i is the kinetic energy and V_{ij} a nucleon-nucleon interaction [**?**].

The Schrödinger equation associated with this Hamiltonian can not be solved exactly when $A > 3$. For very light systems ($A \sim 4-5$) efficient methods [**?**] exist, even for continuum states [**?**]. However most reactions relevant in nuclear astrophysics involve heavier nuclei essentially with nucleon or α projectiles. Recent developments of *ab initio* models (see for example Refs. [**?**, **?**, **?**]) are quite successful for spectroscopic properties of low-lying states. These models make use of realistic interactions, fitted on many properties of the nucleon-nucleon system. A recent work within the Fermionic Molecular Dynamics [**?**] succeeded in computing the ^{3}He$(\alpha,\gamma)^7$Be cross section from a realistic interaction, and without the cluster approximation. However, a consistent description of bound and scattering states of an A-body problem remains a very difficult task [**?**], in particular for transfer reactions.

In cluster models, it is assumed that the nucleons are grouped in clusters [**?**, **?**]. We present here the specific application to two-cluster systems. The internal wave functions of the clusters are denoted as $\phi_i^{I_i\pi_i\nu_i}(\xi_i)$, where I_i and π_i are the spin and parity of cluster i, and ξ_i represents a set of their internal coordinates.

The wave function in channel c is written by using the antisymetrization operator $\mathcal{A}$ as

$$\Psi_c^{JM\pi} = \mathcal{A}\, g_c^{J\pi}(\rho)\ \varphi_c^{JM\pi}(\Omega_\rho, \xi_1^i, \xi_2^j), \tag{4.18}$$

which corresponds to the Resonating Group (RGM) definition [**?**,**?**,**?**]. In most applications, the internal cluster wave functions $\phi_i^{I_i\pi_i\nu_i}$ are defined in the shell model. Accordingly, the nucleon-nucleon interaction must be adapted to this choice, which leads to effective forces, such as the Volkov [**?**] or the Minnesota [**?**] interactions. The relative wave functions $g_{\alpha\ell I}^{J\pi}(\rho)$ are to be determined from the Schrödinger equation, which is transformed into a integro-differential equation involving a non-local potential [**?**]. In most applications, this relative function is expanded over Gaussian functions [**?**, **?**], which corresponds to the Generator Coordinate Method (GCM). The wave function (4.18) is rewritten as

$$\Psi_c^{JM\pi} = \int f_c^{J\pi}(R)\Phi_c^{JM\pi}(R), \tag{4.19}$$

where $\Phi_c^{JM\pi}(R)$ is a projected Slater determinant, and $f_c^{J\pi}(R)$ the generator function, which must be determined. The GCM is equivalent to the RGM, but is better adapted to numerical calculations, as it makes uses of projected Slater determinants (see Refs. [**?**, **?**] for detail).

The main advantage of cluster models with respect to other microscopic theories is their ability to deal with reactions, as well as with nuclear spectroscopy. The first applications were done for reactions involving light nuclei, such as d, ^{3}He or α particles [**?**, **?**]. More recently, much work has been devoted to the improvement of the internal wave functions:

multicluster descriptions [**?**], large-basis shell model extensions [**?**], or monopolar distortion [**?**].

As mentioned before, the RGM radial wave functions are expanded over a Gaussian basis. The GCM is well adapted to numerical calculations, and to a systematic approach, but the Gaussian behaviour is not physical at large distances, and must be corrected. We use the Microscopic R-matrix Method [**?**,**?**] which is a direct extension of the standard R-matrix technique [**?**], based on the existence of two regions: the internal region (with channel radius a), where the nuclear force and the nucleus-nucleus antisymmetrization are important, and the external region where they can be neglected. In the external region, the Gaussian behaviour of the RGM radial function is replaced by Coulomb functions. Matching the internal and external components provide, either the collision matrix for scattering states, or the binding energy for bound states.

4.2.3. The phenomenological R-matrix method

The R-matrix method is well known in atomic and nuclear physics [**?**]. As mentioned in the previous section, the basic idea is to divide the space in two regions: the internal region (with radius a), where the nuclear force is important, and the external region, where the interaction between the nuclei is governed by the Coulomb force only. Although the R-matrix parameters do depend on the channel radius a, the sensitivity of the cross section with respect to its choice is quite weak. In the R-matrix method, the energy dependence of the cross sections is obtained from Coulomb functions, as expected from the Schrödinger equation.

The physics of the internal region is determined by a number N of poles, which are characterized by energy E_λ and reduced widths $\gamma_{\lambda i}$. In a multichannel problem, the R-matrix at energy E is defined as

$$R_{if}^{J\pi}(E) = \sum_{\lambda=1}^{N} \frac{\gamma_{\lambda i}\gamma_{\lambda f}}{E_\lambda - E}, \tag{4.20}$$

which must be given for each partial wave $J\pi$ (these indices are dropped in the R-matrix parameters for the sake of clarity). Indices i and f refer to the initial and final channels. The pole properties are associated with the physical energy and width of resonances, but not strictly equal. This is known as the difference between “formal" and “observed" parameters, deduced from experiment. In a general case, involving more than one pole, the link between those two sets is not straightforward (see Refs. [**?**, **?**] for a general formulation of this problem).

The method can be applied in two ways: (i) in the *calculable* R-matrix, parameters E_λ, $\gamma_{\lambda i}$ and $\gamma_{\lambda f}$ are obtained from a variational calculation; (ii) in the *phenomenological* R-matrix variant, these quantities are fitted to experiment. The calculable R-matrix method is used, for example in microscopic calculations (see section 4.2.2.). Many applications exist in atomic and in nuclear physics.

Although the origin of the phenomenological variant is identical, its application is somewhat different. In nuclear astrophysics the main goal of the R-matrix method [**?**] is to parameterize some experimentally known quantities, such as cross sections or phase shifts, with a small number of parameters, which are then used to extrapolate the cross section

down to astrophysical energies. A well known example is the $^{12}C(\alpha,\gamma)^{16}O$ reaction, which has been studied by many groups. In spite of impressive experimental efforts, the lowest experimental energies are around 0.8 MeV, whereas the Gamow peak (at the typical He-burning temperature $T_9 = 0.2$) is $E_0 \approx 0.3$ MeV. At these subcoulomb energies, the cross sections drop by several orders of magnitude, and extrapolation techniques are necessary. We refer to Refs. [**?**] for recent works on this topic.

The R-matrix method can be applied to transfer as well as to capture reactions. It is usually used to investigate resonant reactions but is also suited to describe non-resonant processes [**?**]. In the latter case, the non-resonant behavior is simulated by a high-energy pole, referred to as the background contribution, which makes the R-matrix nearly energy independent.

4.2.4. The DWBA method

The Distorted Wave Born Approximation (DWBA) starts from the premise that elastic scattering is dominant and has to be treated fully, while non-elastic events can be treated by perturbation theory. Although DWBA is a first-order theory, the way it is usually applied is not simply first-order. That is because optical potentials fitted to elastic scattering data may include higher-order effects implicitly. Therefore different potentials are needed for higher-order methods, such as coupled-channels calculations, than those used in DWBA in order to reproduce the same elastic data.

The DWBA method can be applied to transfer reactions

$$a(=b+x)+A \rightarrow b+B(=A+x) \tag{4.21}$$

and assumes that particle x goes from the projectile a to the target A [**?**]. Typical examples are (d,p) and (d,n) reactions, where a nucleon is transferred from the incident deuteron to the target. The cross section for the transfer reaction (4.21) is obtained from the matrix elements

$$T^{\rm DWBA} = \iint dr_\alpha dr_\beta\, g_\beta(r_\beta) < \Psi_b \Psi_B |\Delta V| \Psi_a \Psi_A > g_\alpha(r_\alpha). \tag{4.22}$$

The distorted waves $g_\alpha(r_\alpha)$ and $g_\beta(r_\beta)$, (r_α and r_β are the relative coordinates) corresponding to the relative motion in the entrance and exit channels, respectively, are generated by optical potentials U_α and U_β (see Refs. [**?**, **?**] for more detail). For the sake of clarity we omit the quantum numbers in the radial functions. The residual interaction is defined in two different ways

$$\begin{aligned} \Delta V &= V_{xA} + V_{bA} - U_\alpha \ \ \text{(prior)} \\ &= V_{bx} + V_{bA} - U_\beta \ \ \text{(post)}, \end{aligned} \tag{4.23}$$

which correspond to "prior" and "post" definitions, respectively; they provide identical values for T^{DWBA}. The main problem of the method is that the potentials are usually poorly known. In general, a good approximation is to neglect $V_{bA} - U_\alpha$ or $V_{bA} - U_\beta$. The matrix element then contains distorted scattering wave functions χ_α, χ_β, and the radial bound state wave functions of the transferred cluster.

Since more realistic descriptions of nucleus $a(B)$ should involve other configurations than $b+x$ $(A+x)$, spectroscopic factors are introduced ($\mathcal{S}_a$ and $\mathcal{S}_B$). The DWBA cross section is therefore linked to the experimental cross section through

$$\sigma_{\rm exp} = \mathcal{S}_a \mathcal{S}_B \sigma_{\rm DWBA}. \tag{4.24}$$

The precision of the DWBA has been investigated in Ref. [**?**] for transfer reactions at low energies. The ^{13}C(α,n)^{16}O cross section was calculated in a microscopic model, and in the DWBA with conditions as close as possible to the reference calculation. The conclusion is twofold. On one hand, the DWBA method turns out to be very sensitive to the conditions of the calculations: choice of the nucleus-nucleus potentials and, to a lesser extent, of the internal wave functions of the colliding nuclei. This sensitivity is due to very basic properties, i.e. the short-range character of the DWBA matrix elements, which are quite sensitive to details of the wave functions. On the other hand, the difference between the DWBA and the reference microscopic method can be fairly large, and varies with angular momentum. This is most likely due to antisymmetrization effects which are approximately included in the DWBA through the choice of deep nucleus-nucleus potentials. This property should also occur in other systems and suggests that the DWBA method can only provide transfer cross sections with a non-negligible uncertainty.

4.3. Cross sections

4.3.1. Transfer reactions

Although elastic scattering does not play any role in the nucleosynthesis, it represents the simplest process. It is also used to determine the properties of low-energy resonances (see for example Ref [**?**]). For the sake of simplicity, we limit ourselves to single-channel calculations involving zero-spin nuclei. In that case the collision matrix $U^{J\pi}$ contains a single element, parameterized as

$$U^\ell = \exp(2i\delta^\ell), \tag{4.25}$$

and the scattering amplitude is given by

$$f(\theta) = \frac{1}{2ik}\sum_\ell (2\ell+1)(U^\ell - 1)P_\ell(\cos\theta), \tag{4.26}$$

where $P_\ell(x)$ is a Legendre polynomial. For real potentials, δ^ℓ is real and $|U^\ell| = 1$. Absorption can be simulated by an imaginary part of the potential but (4.26) remains valid. The elastic cross section is obtained from

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \tag{4.27}$$

In charged-particle systems, series (4.26) converges very slowly owing to the long range of the Coulomb interaction. In these conditions, the phase shift δ^ℓ and the scattering amplitude $f(\theta)$ are divided in two parts: the nuclear and Coulomb contributions. This gives

$$\begin{aligned} \delta^\ell &= \delta^\ell_N + \delta^\ell_C \\ f(\theta) &= f_N(\theta) + f_C(\theta), \end{aligned} \tag{4.28}$$

where the Coulomb phase shift δ_C^ℓ and scattering amplitude $f_C(\theta)$ are known analytically. The nuclear contribution $f_N(\theta)$ converges rapidly (see Ref. [**?**] for detail).

In multichannel calculations, the collision matrix $U^{J\pi}$ contains several elements (number of open channels c). The non-diagonal terms provide, either inelastic cross sections, or transfer cross sections. The integrated cross section for a transition from the entrance channel 0 to a channel α is given by

$$\sigma_t(0\to\alpha)=\frac{\pi}{k_0^2}\frac{1}{(2I_1+1)(2I_2+1)}\sum_{J\pi}(2J+1)\sum_{\ell_0 I_0 \ell_\alpha I_\alpha}|U^{J\pi}_{0\ell_0 I_0,\alpha\ell_\alpha I_\alpha}|^2, \tag{4.29}$$

where I_1 and I_2 are the spins of the colliding nuclei. At low energies, relevant in nuclear astrophysics, the summation (4.29) contains very few terms. In particular, resonant reactions often involve a single term. The differential cross section can be found, for example in Refs. [**?**, **?**].

4.3.2. Radiative capture

Radiative capture reactions play an important role in the nucleosynthesis [**?**]. They can be seen as a transition from an initial scattering state to a final bound state of the system. This process arises from the electromagnetic interaction, and can therefore be treated in the perturbation theory [**?**]. If H_e is the photon-emission Hamiltonian for the nuclear system, the capture cross section to a final state $J_f\pi_f$ is given by

$$\begin{aligned}\frac{d\sigma_c}{d\Omega_\gamma}(E,J_f\pi_f) &= \frac{k_\gamma}{2\pi\hbar}\frac{1}{(2I_1+1)(2I_2+1)}\\ &\times\sum_{q\nu_1\nu_2 M_f}|<\Psi^{J_fM_f\pi_f}|H_e(q,\Omega_\gamma)|\Psi^{\nu_1\nu_2}_{0k}>|^2,\end{aligned} \tag{4.30}$$

where $q=\pm1$ is the photon polarization, $\Psi^{J_fM_f\pi_f}$ the final-state wave function, and where we have explicitly written the spin orientations $\nu_1\nu_2$.

To compute the cross section, the initial wave function is expanded in partial waves, and the electromagnetic operator is expanded in multipoles $\mathcal{M}^\sigma_{\lambda\mu}$. The electric $\sigma=E$ and magnetic $\sigma=M$ multipole operators are given by

$$\begin{aligned}\mathcal{M}^E_{\lambda\mu}&=e\sum_i^A g_\ell(i)\,r_i^\lambda Y^\mu_\lambda(\Omega_{r_i})\\ \mathcal{M}^M_{\lambda\mu}&=\mu_N\sum_i^A\left[\frac{2}{\lambda+1}g_\ell(i)\ell_i+g_s(i)s_i\right]\cdot\nabla r_i^\lambda Y^\mu_\lambda(\Omega_{r_i}),\end{aligned} \tag{4.31}$$

with

$$\begin{aligned}g_\ell(i)&=1/2-t_{iz})\\ g_s(i)&=g_p(1/2-t_{iz})+g_n(1/2+t_{iz})\end{aligned} \tag{4.32}$$

where t_{iz} is the isospin projection of nucleon i, and g_p, g_n are the gyromagnetic factors of the proton ($g_p=5.586$) and of the neutron ($g_n=-3.826$). Equations (4.31) are written in

the framework of a microscopic approach. Simplified expressions, valid for two-particles can be found, for example in Refs. [**?**, **?**]. After integration over the photon angle Ω_γ, the total cross section is given by

$$\begin{aligned}\sigma_c(E,J_f\pi_f) &= \frac{2J_f+1}{(2I_1+1)(2I_2+1)} \\ &\times \sum_{\sigma\lambda J_i I\ell_i} \frac{k_\gamma^{2\lambda+1}}{2\ell_i+1}\frac{8\pi(\lambda+1)}{\hbar\lambda(2\lambda+1)!!^2}| < \Psi^{J_f\pi_f}||\mathcal{M}^\sigma_\lambda||\Psi^{J_i\pi_i}_{\ell_0 I_0}(E) > |^2, \quad (4.33)\end{aligned}$$

where ℓ_0 and I_0 refer to the entrance channel. The summations in (4.33) are limited by the usual selection rules

$$\begin{aligned}&|J_i - J_f| \leq \lambda \leq J_i + J_f \\ &\pi_i\pi_f = (-)^\lambda \text{ (for } \sigma = E), \\ &\pi_i\pi_f = (-)^{\lambda+1} \text{ (for } (\sigma = M). \end{aligned} \quad (4.34)$$

In addition, the long-wavelength approximation ($k_\gamma R \ll 1$, where R is a typical dimension of the system) strongly reduces the summation over λ. In general, a single multipole is important. As in the previous subsection we only give the integrated cross section, where no interference between multipoles and partial waves shows up. In contrast, the differential cross section (4.30) involves interference terms (see definition in Ref. [**?**]).

4.3.3. Fusion

5. Experimental measurements

5.1. Direct methods

5.2. Indirect

(Coulomb breakup, Trojan horse, ANC, spectroscopic factors)

5.3. Radioactive beams

5.4. Underground experiments

5.5. electron screening

6. Recent works on important reactions

He3(a,g), Be7(p,g), Li8(p,a), c12(a,g), f18(p,a) autres?????

7. Conclusion

here is [**?**]

References